Lectures on Monetary Policy, Inflation and the Business Cycle

A Model with Sticky Wages and Prices

by

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Based on: Erceg, Henderson and Levin. (JME, 2000)

**Firms**

*Technology*

\[ Y_t(i) = A_t \, N_t(i)^{1-\alpha} \]

\[ N_t(i) \equiv \left[ \int_0^1 N_t(i, j)^{1-\frac{1}{\epsilon_w}} \, dj \right]^{\frac{\epsilon_w}{\epsilon_w-1}} \]

**Cost minimization:**

\[ N_t(i, j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \tag{1} \]

for all \( i, j \in [0, 1] \), where

\[ W_t \equiv \left[ \int_0^1 W_t(j)^{1-\epsilon_w} \, dj \right]^{\frac{1}{1-\epsilon_w}} \]

In addition,

\[ \int_0^1 W_t(j)N_t(i, j) \, dj = W_tN_t(i). \]
Optimal price setting (as in baseline sticky price model)

\[
\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k \ E_t \left\{ Q_{t,t+k} \ (P_t^* \ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})) \right\}
\]

subject to

\[
Y_{t+k|t} = (P_t^*/P_{t+k})^{-\epsilon_p} \ C_{t+k}
\]

Aggregation:

\[
\pi_t^p = \beta \ E_t\{\pi_{t+1}^p\} - \lambda_p \ \hat{\mu}_t^p
\]  \hspace{1cm} (2)

where \(\hat{\mu}_t^p \equiv \mu_t^p - \mu^p \equiv -\hat{m} c_t\), \(\mu^p \equiv \log \frac{\epsilon_p}{\epsilon_p - 1}\), and \(\lambda_p \equiv \frac{(1-\theta_p)(1-\beta \theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha + \alpha \epsilon_p}\).
Households

• fraction of households/trade unions adjusting nominal wage: \( 1 - \theta_w \)
• \( \theta_w \): index of nominal wage rigidity

Optimal Wage Setting

\[
\max \sum_{k=0}^{\infty} (\beta \theta_w)^k E_t\{U(C_{t+k|t}, N_{t+k|t})\}
\]

subject to:

\[
N_{t+k|t} = (W_t^*/W_{t+k})^{-\epsilon_w} N_{t+k}
\]

\[
P_{t+k} C_{t+k|t} + E_t Q_{t+k, t+k+1} D_{t+k+1|t} \leq D_{t+k|t} + W_t^* N_{t+k|t} - T_{t+k}
\]

where \( N_t \equiv \int_0^1 N_t(i) \, di \).
Optimality condition:

$$\sum_{k=0}^{\infty} (\beta \theta_w)^k N_{t+k|t} E_t \left\{ U_c(C_{t+k|t}, N_{t+k|t}) \frac{W_t^*}{P_{t+k}} + \mathcal{M}_w U_n(C_{t+k|t}, N_{t+k|t}) \right\} = 0$$

where $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$

*Complete markets:* $C_{t+k|t} = C_{t+k}$ for $k = 0, 1, 2, \ldots$

Letting $MRS_{t+k|t} \equiv -\frac{U_n(C_{t+k|t}, N_{t+k|t})}{U_c(C_{t+k|t}, N_{t+k|t})}$

$$\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ N_{t+k|t} U_c(C_{t+k|t}, N_{t+k|t}) \left( \frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0$$

(3)
Full wage flexibility ($\theta_w = 0$):

$$\frac{W^*_t}{P_t} = \frac{W_t}{P_t} = M_{w,MRS_{t|t}}$$

Zero inflation steady state:

$$\frac{W^*}{P} = M_{w,MRS}$$
Log-linearization (after dividing (3) by $Mw$ MRS):

$$
w_t^* = \mu^w + (1 - \beta \theta_w) \sum_{k=0}^{\infty} (\beta \theta_w)^k \ E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \quad (4)
$$

where $\mu^w \equiv \log \frac{\epsilon_w}{\epsilon_w - 1}$.

With isoelastic separable utility $\implies mrs_{t+k|t} = \sigma \ c_{t+k} + \varphi \ n_{t+k|t}$.

Average marginal rate of substitution: \quad $mrs_{t+k} \equiv \sigma \ c_{t+k} + \varphi \ n_{t+k}$

$$
mrs_{t+k|t} = mrs_{t+k} + \varphi \ (n_{t+k|t} - n_{t+k})
\quad = mrs_{t+k} - \epsilon_w \varphi \ (w_t^* - w_{t+k})
$$
Hence,

\[ w^*_t = \frac{1 - \beta \theta_w}{1 + \epsilon_w \varphi} \sum_{k=0}^{\infty} (\beta \theta_w)^k \ E_t \left\{ \mu_w + mrs_{t+k} + \epsilon_w \varphi \ w_{t+k} + p_{t+k} \right\} \]

\[ = \frac{1 - \beta \theta_w}{1 + \epsilon_w \varphi} \sum_{k=0}^{\infty} (\beta \theta_w)^k \ E_t \left\{ (1 + \epsilon_w \varphi) \ w_{t+k} - \hat{\mu}_w \right\} \]

where \( \hat{\mu}_t^w \equiv \mu_t^w - \mu^w \)

More compactly:

\[ w^*_t = \beta \theta_w \ E_t\{w^*_{t+1}\} + (1 - \beta \theta_w) \ (w_t - (1 + \epsilon_w \varphi)^{-1} \ \hat{\mu}_t^w) \] (5)
Wage Inflation Dynamics

\[ W_t = \left[ \theta_w W_{t-1}^{1-\epsilon_w} + (1 - \theta_w) W_t^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}} \]

Log-linearization:

\[ w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^* \]  \hspace{1cm} (6)

Combining (5) and (6):

\[ \pi_t^w = \beta E_t \{ \pi_{t+1}^w \} - \lambda_w \hat{\mu}_t^w \]  \hspace{1cm} (7)

where \( \lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w)} \).

Additional Optimality Condition

\[ c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1}^p \} - \rho) \]
Equilibrium

Define real wage gap:
\[ \tilde{\omega}_t \equiv \omega_t - \omega_t^n \]

Price markups vs. output and real wage gaps:
\[ \hat{\mu}_t^p = (mpn_t - \omega_t) - \mu_t^p \]
\[ = (\bar{y}_t - \bar{n}_t) - \tilde{\omega}_t \]
\[ = -\frac{\alpha}{1 - \alpha} \bar{y}_t - \tilde{\omega}_t \] (8)

Combining (2) and (8):
\[ \pi_t^p = \beta \ E_t \{ \pi_{t+1}^p \} + \kappa_p \bar{y}_t + \lambda_p \tilde{\omega}_t \] (9)

where \( \kappa_p \equiv \frac{\alpha \lambda_p}{1 - \alpha} \).
Wage markups vs. output and real wage gaps:

\[
\tilde{\mu}_t^w = \omega_t - mrs_t - \mu^w \\
= \tilde{\omega}_t - (\sigma \tilde{y}_t + \varphi \tilde{n}_t) \\
= \tilde{\omega}_t - \left( \sigma + \frac{\varphi}{1 - \alpha} \right) \tilde{y}_t
\]  (10)

Combining (7) and (10):

\[
\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t
\]  (11)

where \( \kappa_w \equiv \lambda_w \left( \sigma + \frac{\varphi}{1 - \alpha} \right). \)
Wage gap identity:

\[ \tilde{\omega}_{t-1} \equiv \tilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta \omega_t^n \quad (12) \]

**Dynamic IS equation**

\[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\tilde{y}_{t+1}\} \quad (13) \]

**Interest Rate Rule:**

\[ i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + v_t \quad (14) \]
Dynamical system:

\[ x_t = A_w \ E_t\{x_{t+1}\} + B_w \ z_t \]  \quad (15)

where

\[ x_t \equiv [\tilde{y}_t, \, \pi_t^p, \, \pi_t^w, \, \tilde{\omega}_{t-1}]' \]
\[ z_t \equiv [\tilde{r}_t^n - n_t, \, \Delta \omega_t^n]' \]

Remark: \( \tilde{y}_t = \pi_t^p = \pi_t^w = 0 \) cannot be solution, unless \( \omega_t^n \) is constant.

Conditions for uniqueness of the equilibrium

Particular case \((\phi_y = 0)\):

\[ \dot{\phi}_p + \dot{\phi}_w > 1 \]
Dynamic Responses to a Monetary Policy Shock

*Interest rate rule:* \( \phi_p = 1.5, \; \phi_y = \phi_w = 0, \; \rho_v = 0.5 \)

*Three calibrations:*

**Baseline:** \( \theta_p = 2/3, \; \theta_w = 3/4 \)

**Flexible wage:** \( \theta_p = 2/3, \; \theta_w = 0 \)

**Flexible price:** \( \theta_p = 0, \; \theta_w = 3/4 \)

Figure 6.3
Monetary Policy Design with Sticky Wages and Prices

Second Order Approximation to Welfare Losses

\[
\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w(1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right) + t.i.p.
\]

\[
\mathbb{L} = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \text{var}(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p} \text{var}(\pi_t^p) + \frac{\epsilon_w(1 - \alpha)}{\lambda_w} \text{var}(\pi_t^w)
\]

Key policy issues

- replicating the natural equilibrium allocation is generally unfeasible.
- optimal monetary policy
- evaluation of alternative simple rules
Optimal Monetary Policy

\[
\min E_0 \sum_{t=0}^{\infty} \beta^t \left( \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi^p_t)^2 + \frac{\epsilon_w(1 - \alpha)}{\lambda_w} (\pi^w_t)^2 \right)
\]

subject to

\[
\pi^p_t = \beta E_t \{ \pi^p_{t+1} \} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t
\]

\[
\pi^w_t = \beta E_t \{ \pi^w_{t+1} \} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t
\]

\[
\tilde{\omega}_{t-1} \equiv \tilde{\omega}_t - \pi^w_t + \pi^p_t + \Delta \omega^*_t
\]
Optimality conditions:

\[
\left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t + \kappa_p \xi_{1,t} + \kappa_w \xi_{2,t} = 0 \tag{16}
\]

\[
\frac{\epsilon_p}{\lambda_p} \pi_t^p - \Delta \xi_{1,t} + \xi_{3,t} = 0 \tag{17}
\]

\[
\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \pi_t^w - \Delta \xi_{2,t} - \xi_{3,t} = 0 \tag{18}
\]

\[
\lambda_p \xi_{1,t} - \lambda_w \xi_{2,t} + \xi_{3,t} - \beta E_t \{ \xi_{3,t+1} \} = 0 \tag{19}
\]
Combined with (9), (11), and (12):

\[ A_0^* x_t = A_1^* E_t\{x_{t+1}\} + B^* \Delta a_t \]

where \( x_t \equiv [\tilde{y}_t, \pi_t^p, \pi_t^w, \tilde{\omega}_{t-1}, \xi_{1,t-1}, \xi_{2,t-1}, \xi_{3,t}]' \)

*Dynamic Responses to a Technology Shock* (Figure 6.4)
A Special Case with an Analytical Solution

Define:

\[ \pi_t \equiv (1 - \vartheta) \pi_t^p + \vartheta \pi_t^w \]  

(20)

where \( \vartheta \equiv \frac{\lambda_p}{\lambda_p + \lambda_w} \in [0, 1] \)

Note that (9) and (11) imply:

\[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \]  

(21)

where \( \kappa \equiv \frac{\lambda_p \lambda_w}{\lambda_p + \lambda_w} \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \)

- no trade-off!
- when is it optimal to fully stabilize \( \pi_t \) (and the output gap)?
Assumptions: \( \kappa_p = \kappa_w \); \( \epsilon_p = \epsilon_w (1 - \alpha) \equiv \epsilon \)

Then, (16)-(18) simplify to:

\[
\lambda_w \pi_t^p + \lambda_p \pi_t^w = -\frac{\lambda_p}{\epsilon} \Delta \tilde{y}_t
\]

for \( t = 1, 2, 3, \ldots \) and \( \lambda_w \pi_0^p + \lambda_p \pi_0^w = -\frac{\lambda_p}{\epsilon} \tilde{y}_0 \) for period 0

Equivalently,

\[
\pi_t = -\frac{\vartheta}{\epsilon} \Delta \tilde{y}_t
\]

for \( t = 1, 2, 3, \ldots \), and \( \pi_0 = -\frac{\vartheta}{\epsilon} \tilde{y}_0 \) in period 0.

In levels:

\[
\hat{q}_t = -\frac{\vartheta}{\epsilon} \tilde{y}_t \tag{22}
\]

where \( \hat{q}_t \equiv q_t - q_{-1} \), and \( q_t \equiv (1 - \vartheta) p_t + \vartheta w_t \).
Combining (22) and (21) (using $\pi_t \equiv \hat{q}_t - \hat{q}_{t-1}$):

$$
\hat{q}_t = a \hat{q}_{t-1} + a\beta E_t\{\hat{q}_{t+1}\} = 0
$$
for $t = 0, 1, 2, \ldots$ where $a \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa \epsilon}$.

Stationary solution:

$$
\hat{q}_t = \delta \hat{q}_{t-1}
$$
where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$ for $t = 0, 1, 2, \ldots$

Given that $\hat{q}_{-1} = 0$, the optimal policy requires:

$$
\pi_t = 0
$$
$$
\tilde{y}_t = 0
$$
for $t = 0, 1, 2, \ldots$
Evaluation of Simple Rules under Sticky Wages and Prices

Six rules:

- strict price inflation targeting ($\pi^p_t = 0$, all $t$)
- strict wage inflation targeting ($\pi^w_t = 0$, all $t$)
- strict composite inflation targeting ($\pi_t = 0$, all $t$)
- flexible price inflation targeting ($i_t = \rho + 1.5 \pi^p_t$)
- flexible wage inflation targeting ($i_t = \rho + 1.5 \pi^w_t$)
- flexible composite inflation targeting ($i_t = \rho + 1.5 \pi_t$)

Three scenarios

- baseline: $\theta_p = 2/3$ ; $\theta_w = 3/4$
- low wage rigidities: $\theta_p = 2/3$ and $\theta_w = 1/4$
- low price rigidities: $\theta_p = 1/3$ and $\theta_w = 3/4$
### Table 6.1: Evaluation of Simple Rules

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<th>Optimal Policy</th>
<th>Strict Rules</th>
<th>Flexible Rules</th>
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<td>Price</td>
<td>Wage</td>
<td>Composite</td>
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<td>$\sigma(\pi^w)$</td>
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<td>$\sigma(\tilde{y})$</td>
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