In this paper we empirically examine the sources of the volatility of the foreign exchange risk premia. Using a nonlinear structural Vector Autoregression (VAR) model based on no-arbitrage condition to identify various macroeconomic shocks and the foreign exchange risk premia, we find that more than 80% of the volatilities of the currency risk premia can be accounted for by the standard macroeconomic shocks that drive output and inflation. By explicitly modelling the currency risk premia in the VAR system, we also offer a potential reconciliation for the seemingly contradicting observations from the previous VAR analysis of the exchange rate “overshooting” behavior under exogenous monetary innovations.

Keywords: Macroeconomic Shocks; Foreign Exchange Risk Premia; Nonlinear VAR; Exchange Rate Overshooting

1. INTRODUCTION

One of the long-lasting puzzles in international finance is the forward premium anomaly in currency markets. This refers to the well-documented empirical phenomenon (e.g., Hodrick, 1987) that the domestic currency tends to appreciate when domestic nominal interest rates exceed foreign interest rates, a clear violation of uncovered interest parity (henceforth UIP). A deviation from UIP often is interpreted as the risk premium from investing in foreign currency by rational and risk-averse investors in the standard international monetary asset pricing models (e.g., Lucas, 1982).\(^1\) Stylized facts about the foreign exchange risk premia are that they are not only negatively correlated with the subsequent depreciation of the foreign currency but also are extremely volatile (Fama, 1984). Reasonably parameterized consumption-and-money based dynamic asset pricing models, however, fail to generate the risk premia that are volatile enough to match the empirical evidence (see Backus et al., 1993; Bekaert, 1996; Bekaert et al., 1997; among others).\(^2\) Engel (1996) provides an excellent survey of this literature.

We would like to thank an associate editor and an anonymous referee for helpful comments and constructive suggestions. All remaining errors are ours. Address correspondence to: Shigeru Iwata, Department of Economics, The University of Kansas, 213 Summerfield Hall, Lawrence, KS 66045, USA; e-mail: iwata@ku.edu (Shigeru Iwata) and shuwu@ku.edu (Shu Wu).
This failure prompts alternative theoretical structural models that attempt to reconcile with the contradicting stylized facts, but so far there has been little apparent breakthrough in those models. To address the issue in this paper, we take an intermediate approach between a fully structural model and pure data summarization. We focus on understanding the sources of the volatility of the foreign exchange risk premia empirically. Are the volatilities of the currency risk premia mostly due to the fundamental macroeconomic shocks that are standard inputs to the international asset pricing models? Or are they rather some exogenous shocks to the foreign exchange market? Understanding the impact of these different shocks is important when constructing a structural model that would be consistent with the stylized facts.

More specifically, we start with the structural relation between the exchange rate change and the interest rate differential derived from the no arbitrage condition in international financial markets. We then connect the deviation from UIP in this relation to the standard macroeconomic shocks identified on the basis of the structural VAR models frequently used in empirical work (e.g., Christiano, Eichenbaum, and Evans, 1999). The resulting system is a nonlinear structural VAR model. The dynamic effects of those exogenous shocks on the risk premia are then examined. We find that most of the volatility (more than 80% on average) of the foreign exchange risk premia are explained by the standard macroeconomic shocks that drive output and inflation. The exogenous shock to the exchange rate, by contrast, which has little impact on macroeconomic fundamentals, only accounts for a small fraction of the volatilities of the currency risk premia. This result may provide some discipline on the channels through which economic shocks can affect currency risk premia. It is not an external exchange rate shock but the standard macroeconomic shocks in the two countries that drive the risk premia on returns from currency speculation.

Hollifield and Yaron (2000) use a similar no-arbitrage based approach to decompose the foreign exchange risk premia into real and nominal components. They found that most of variations in the risk premia are driven by the real factor. The difference between the current paper and Hollifield and Yaron (2000) is that we incorporate the no-arbitrage based asset-pricing relation into a structural VAR model driven by various fundamental macroeconomic shocks.

The current paper is also closely related to the empirical literature of VAR analysis of the monetary policy effect on the exchange rate. The main focus of these studies is on the overshooting phenomenon (Dornbush, 1976) of the exchange rate in response to a monetary policy shock. Clarida and Gali (1994), Eichenbaum and Evans (1995), and Grilli and Roubini (1996) find strong evidence of the “delayed overshooting” of the exchange rate. Cushman and Zha (1997) and Faust and Rogers (2003), however, obtain the results showing no delayed overshooting.

Our approach differs from these VAR studies in that we explicitly model the time-varying foreign exchange risk premia and, by so doing, introduce a nonlinear structure into the VAR system. Because the risk premium is very volatile, a large fraction of the exchange rate movement must be attributed to the changes in the
risk premium. Therefore, how the exchange rate responses to monetary policy shocks may depend critically on the dynamic properties of the risk premia.

We find that the risk premium increases significantly in response to an expansionary monetary policy shock, which is consistent with the previous VAR studies that find large UIP deviations following monetary innovations. Because our VAR is nonlinear, the model can capture the state-dependent effects of the monetary innovations. We find large variations in the magnitude of the dynamic response of the currency risk premium across different states of the economy. The “delayed overshooting” of the exchange rate, which often is reported in the literature, would occur if the increase in the risk premium outweighs the decrease in the interest rate under such shocks. If the response of the risk premium is smaller than that of the interest rate, however, the exchange rate exhibits the standard overshooting behavior without any delay. This finding, therefore, offers a potential reconciliation for the seemingly contradicting observations from the previous VAR analysis of the monetary policy effect on the exchange rate.

The rest of the paper is organized as follows. Section 2 lays out the empirical model we use to examine the relation between the foreign exchange risk premia and macroeconomic shocks. Section 3 discusses the main results, and Section 4 contains some concluding remarks.

2. THE MODEL

In this section, we first outline a general relationship between the exchange rate movements, currency risk premia and the short term interest rate differential, based on the no-arbitrage condition widely used in the finance literature. We next incorporate this relation into a structural VAR system that links key economic aggregates and the exchange rate to the fundamental macroeconomic shocks. The resulting nonlinear structural VAR system is then used for analyzing the effects of macroeconomic shocks, including monetary policy shocks, on the foreign exchange risk premia.

2.1. The Foreign Exchange Risk Premia

Absence of arbitrage in asset markets (e.g., Harrison and Kreps 1979) implies that there exists a positive stochastic discount factor \( M_{t+1} \) such that for any asset denominated in domestic currency,

\[
1 = E_t(M_{t+1} R_{t+1}),
\]

where \( R_{t+1} \) is the gross rate of return on a domestic asset between time \( t \) and \( t + 1 \), and expectation is taken with respect to investors’ information set at time \( t \). In various versions of consumption-and-money based asset pricing model developed since Lucas (1982), \( M_{t+1} \) is simply given by \( \beta \frac{MU_{t+1}}{MU_t} \frac{P_t}{P_{t+1}} \), where \( MU_t \) is the marginal utility of consumption and \( P_t \) is the price level. In this case, \( \ln M_{t+1} \) becomes the (inflation adjusted) growth rate of marginal utility.
Now let $S_t$ be the home currency price of one unit of a foreign currency. Then for any asset denominated in the foreign currency that can be purchased by domestic investors, (1) implies:

$$1 = E_t \left[ M_{t+1} \left( \frac{S_{t+1}}{S_t} \right) R^*_{t+1} \right],$$

(2)

where $R^*_{t+1}$ is the gross rate of return in terms of the foreign currency. But for foreign investors, absence of arbitrage implies that there also must exist a foreign stochastic discount factor satisfying:

$$1 = E_t (M^*_{t+1} R^*_{t+1}).$$

(3)

Therefore, (2) and (3) imply that there exist $M_{t+1}$ and $M^*_{t+1}$ such that:

$$\frac{S_{t+1}}{S_t} = \frac{M^*_{t+1}}{M_{t+1}},$$

(4)

or, in terms of logarithms,

$$\ln S_{t+1} - \ln S_t = -(\ln M_{t+1} - \ln M^*_{t+1}).$$

(5)

This relation is an implication of the general no-arbitrage condition and summarizes the connection between the stochastic discount factors and currency prices. See Backus et al. (2001) and Brandt et al. (2006) for a formal statement of the relation and more detailed derivations.

To get a useful expression for foreign exchange risk premia, we assume that $M_{t+1}$ and $M^*_{t+1}$ both follow the log-normal distribution. More specifically, it is assumed that:

$$M_{t+1} = \exp(\mu_t - \lambda_t^\prime \varepsilon_{t+1})$$

(6)

$$M^*_{t+1} = \exp(\mu^*_t - \lambda^*_t \varepsilon_{t+1}),$$

(7)

where $\mu_t$ and $\mu^*_t$ are scalars, and $\lambda_t$ and $\lambda^*_t$ are two vectors to be specified below. The term $\varepsilon_t$ stands for a vector of fundamental economic shocks distributed as $\mathcal{N}(\mathbf{0}, \mathbf{I})$, including shocks to domestic and foreign monetary policies. And $\lambda_t$ and $\lambda^*_t$ are usually referred to in the literature as the market prices of risk, which we will discuss in detail later.

To see how the exchange rate is related to the interest rates and the market price of risk, let $i_t$ and $i^*_t$ be the continuously compounded short-term interest rates in the home and foreign country, respectively. Then (1) and (3) implies that:

$$i_t = -\ln(E_t M_{t+1})$$

(8)

$$i^*_t = -\ln(E_t M^*_{t+1}).$$

(9)
Using the log-normal assumptions (6) and (7), we can express $\mu_t$ and $\mu_t^*$ as:

$$\mu_t = - \left( i_t + \frac{1}{2} \lambda_t' \lambda_t \right)$$  \hspace{1cm} (10)

$$\mu_t^* = - \left( i_t^* + \frac{1}{2} \lambda_t^* \lambda_t^* \right),$$  \hspace{1cm} (11)

which together with (5) implies:

$$\Delta \ln S_{t+1} = (i_t - i_t^*) + \frac{1}{2} (\lambda_t' \lambda_t - \lambda_t^* \lambda_t^*) + (\lambda_t - \lambda_t^*)' \varepsilon_{t+1}. \hspace{1cm} (12)$$

Note that if $M_{t+1}$ and $M_{t+1}^*$ are not distributed as log-normal, this result still hold as the second-order approximation to (5), as shown in Backus et al. (2001).

It is easy to see from (12) that the conventional uncovered interest parity (UIP) does not hold in general, or:

$$\phi_t \equiv E_t \Delta \ln S_{t+1} - (i_t - i_t^*) \neq 0, \hspace{1cm} (13)$$

where the UIP deviation $\phi_t$ can be expressed as a quadratic function of the home and foreign country’s market prices of risks:

$$\phi_t = \frac{1}{2} (\lambda_t' \lambda_t - \lambda_t^* \lambda_t^*). \hspace{1cm} (14)$$

We may decompose $\phi_t$ as:

$$\phi_t = u_t + v_t, \hspace{1cm} (15)$$

where

$$u_t = (\lambda_t - \lambda_t^*)' \lambda_t \hspace{1cm} (16)$$

$$v_t = - \frac{1}{2} (\lambda_t - \lambda_t^*)' (\lambda_t - \lambda_t^*). \hspace{1cm} (17)$$

Note that, using equation (6) and (12), $u_t$ can be expressed as:

$$u_t = \text{Cov}_t [\Delta \ln S_{t+1} - (i_t - i_t^*), - \ln M_{t+1}], \hspace{1cm} (18)$$

that is, $u_t$ is the conditional covariance between the excess return on the foreign exchange and the log of the stochastic discount factor and hence, is equal to the risk premium from investing in the foreign currency. By (6), we can write $u_t$ as:

$$u_t = \sum_{i=1}^{N} \lambda_{i,t} \cdot \text{Cov}_t [\Delta \ln S_{t+1} - (i_t - i_t^*), \varepsilon_{i,t+1}], \hspace{1cm} (19)$$

which explains why $\lambda_t$ or $\lambda_t^*$ is called the market price of risk. The $i$th component of $\lambda_t$ prices the covariance between the excess return and the $i$th fundamental economic shock. For example, if $\varepsilon_{i,t+1}$ is an exogenous shock to monetary policy
in the home country, then the risk associated with the policy when investing in
the foreign exchange is characterized by the conditional covariance between the
excess return and the policy shock, and \( \lambda_{i,t} \) is the expected excess rate of return
per unit of such covariance.\(^6\)

The second term \( v_t \) in (15) is simply Jensen’s inequality term resulting from
taking logarithm of the foreign exchange return, or:

\[
v_t = -\frac{1}{2} \text{Var}_t [\Delta \ln S_{t+1} - (i_t - i^*_t)]. \tag{20}\]

This term does not have any economic significance and disappears in a continuous
time setting. However, it is interesting to note that both the conditional volatility
of the exchange rate and the risk premium are determined by the home and foreign
country’s market prices of risks. Because in the finance literature the market price
of risk is routinely treated as time-varying, it is not surprising that movements of
the exchange rate are characterized by stochastic volatilities and time-varying risk
premia.

Finally, note that equation (12) provides a link between the foreign exchange risk
premia and macroeconomic shocks. In the finance literature, the market price of
risk is commonly parameterized as a function of a vector of latent state variables of
low dimension without clear economic interpretations. Instead, in what follows,
we will model \( \lambda_t \) and \( \lambda^*_t \) as functions of observable macroeconomic variables,
which are in turn driven by identified fundamental macroeconomic shocks.

2.2. A Nonlinear VAR Model

We postulate two types of shocks in our analysis. One includes exogenous
innovations to output, inflation, and monetary policies in the home and foreign
countries. The other is an exogenous exchange rate innovation orthogonal to those
macroeconomic shocks.

More specifically, we assume that the \( \varepsilon_t \) in equation (12) has seven components:\(^7\)

\[
\varepsilon_t = (\varepsilon'_{Y,t}, \varepsilon'_{\Pi,t}, \varepsilon'_{M,t}, \varepsilon_{s,t})', \tag{21}\]

where \( \varepsilon_{Y,t} = (\varepsilon_{Y,t}, \varepsilon^*_{Y,t})' \) and \( \varepsilon_{\Pi,t} = (\varepsilon_{\Pi,t}, \varepsilon^*_{\Pi,t})' \) can be thought of as the home
and foreign country’s aggregate supply and demand shocks, respectively, whereas
\( \varepsilon_{M,t} = (\varepsilon_{m,t}, \varepsilon^*_{m,t})' \) represents exogenous shocks to the monetary policies in the
two countries. The last element, \( \varepsilon_{s,t} \), is constructed to be the exogenous innovation
to the exchange rate orthogonal to other macroeconomic shocks.

Let \( \mathbf{z}_t \) be a \( 7 \times 1 \) vector of macroeconomic variables that summarizes the current
state of the economy. We include in \( \mathbf{z}_t \) the home and foreign output growth rates
\((y_t, y^*_t)\), as well as the inflation rates \((\pi_t, \pi^*_t)\) in the two countries. Also included
in \( \mathbf{z}_t \) are the home and foreign country’s monetary policy instruments, or the short
term interest rates, \((i_t, i^*_t)\). The last component of \( \mathbf{z}_t \) is the change of the exchange
rate \( (\Delta \ln S_t) \).
We assume that the market prices of risks are linear functions of $z_t$:

$$
\lambda_t = \Gamma z_t
$$

(22)

$$
\lambda_t^* = \Gamma^* z_t,
$$

(23)

where $\Gamma$ and $\Gamma^*$ are $7 \times 7$ matrices. We further assume that the dynamics of the first 6 components of $z_t$ (denoted by $z_t^+$) can be described by the following reduced-form equation:

$$
z_t^+ = \mu + B_1^+ z_{t-1} + \cdots + B_p^+ z_{t-p} + u_t^+,
$$

(24)

where $z_t = (z_t^+, \Delta \ln S_t)'$, $B_1^+, \ldots, B_p^+$ are $6 \times 7$ matrices and $\mu$ are a $6 \times 1$ vector of constants. The $u_t^+$ stands for a vector of one-step-ahead forecast errors and it is assumed that $u_t^+ \sim N(0, \Sigma)$, where $\Sigma$ is a symmetric positive definite matrix. The error term $u_t^+$ is related to the structural shocks according to:

$$
u_t^+ = C \epsilon_t,
$$

(25)

where $C$ is a $6 \times 7$ matrix. Using (12) together with (22) and (23), the last component of $z_t$ may be written as:

$$
\Delta \ln S_t = (i_{t-1} - i_{t-1}^*) + \frac{1}{2} z_{t-1}' (\Gamma' \Gamma - \Gamma^* \Gamma^*) z_{t-1} + z_{t-1}' (\Gamma - \Gamma^*)' \epsilon_t.
$$

(26)

It is then easily seen that (24) and (26) constitute a constrained nonlinear VAR, on which our empirical analysis will be based. More specifically,

$$
z_t = \mu_{t-1} + B(L) z_{t-1} + u_t,
$$

(27)

where

$$
\mu_{t-1} = \begin{bmatrix} \mu \\ (1/2) z_{t-1}' (\Gamma' \Gamma - \Gamma^* \Gamma^*) z_{t-1} \end{bmatrix}
$$

$$
B(L) = \begin{bmatrix} B_1^+(L) \\ b' \end{bmatrix}
$$

$$
u_t = \begin{bmatrix} C \\ z_{t-1}' (\Gamma - \Gamma^*)' \end{bmatrix} \epsilon_t,
$$

with $B^+(L) = B_1^+ + B_2^+ L + \cdots + B_p^+ L^{p-1}$ and $b = (0, 0, 0, 0, 1, -1, 0)'$.

Notice that the last equation in the above system is nonlinear in $z_{t-1}$ [see also equation (26)] and the lagged values of the last element of $z_t$ feed back into the other equations. Therefore, how $\Delta \ln S_t$ responds dynamically to the shock $\epsilon_t$ depends on the initial value of $z_{t-1}$. And through the feedback effect, the dynamic responses of the other variables in the system are also dependent on the initial value of $z_{t-1}$. This implies that the impulse response functions as well as the results of variance decompositions from the VAR system will all depend on $z_{t-1}$.
2.3. Identification

We impose the following restrictions to identify the macroeconomic shocks. First, we assume that output and price do not respond contemporaneously to shocks on monetary policies in both countries, nor are they affected by the current exogenous shocks to the exchange rate. This assumption is widely used in the monetary VAR literature (e.g., Christiano et al., 1999) and does not appear to be unreasonable when monthly data are used in the study. Second, we assume that the monetary authority in each country does not respond contemporaneously to the other country’s aggregate supply and demand shocks as well as the monetary policy shocks when setting its policy instrument. However, we allow monetary authorities to respond contemporaneously to exogenous innovations to the exchange rate, relaxing the restriction imposed in Eichenbaum and Evans (1995).

These identifying assumptions imply that the matrix $C$ takes the following form:

$$C = \begin{pmatrix} C_{11} & 0 \\ \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 \end{pmatrix},$$

where $C_{11}$ is a $4 \times 4$ matrix, $0$ is a $4 \times 3$ matrix of zeroes, “$0$” indicates the zero restriction and “$\times$” indicates a free parameter. In the following estimation we will further normalize $C_{11}$ to be lower triangular.

The matrices $\Gamma$ and $\Gamma^*$ are not identified without further restrictions. Hence we make the following additional identifying assumptions. First, we assume that home investors and foreign investors price the currency risk in a symmetrical fashion in the sense described in the Appendix. Under this assumption, we have $\Gamma^* = A \Gamma A$, where:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

With this restriction, the last equation in (27) can be expressed as:

$$\Delta \ln S_t = z'_{t-1} A'_1 B_S A_2 z_{t-1} + b' z_{t-1} + (C_S z_{t-1})' \nu_t,$$  \hspace{1cm} (28)

where $B_S$ and $C_S$ are, respectively, $4 \times 3$ and $7 \times 7$ matrices whose elements are to be estimated, $b = (0, 0, 0, 0, 1, -1, 0)'$ as defined in (27), and the matrices $A_1$ and $A_2$ are given in the Appendix. See the Appendix for derivation of (28).
Second, to simplify the expression of matrix $C_S$, another type of symmetric restrictions are imposed. We assume, for example, the contribution of $y_t^*$ to the market price of home output risk is equal in size to the contribution of $y_t$ to the market price of foreign output risk. Under this assumption, matrix $C_S$ takes the form as:

$$C_S = \begin{pmatrix}
C_{11} & 0 & 0 & 0 & 0 & 0 & C_{17} \\
0 & -C_{11} & 0 & 0 & 0 & 0 & C_{17} \\
0 & 0 & C_{33} & 0 & 0 & 0 & C_{37} \\
0 & 0 & 0 & -C_{33} & 0 & 0 & C_{37} \\
0 & 0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -C_{55} \\
C_{17} & C_{17} & C_{37} & C_{37} & C_{57} & C_{57} & 0
\end{pmatrix}.$$

See the Appendix for more details.

3. RESULTS

The data used in this study are monthly observations on industrial production, consumer price index (CPI), the short-term interest rate, and the foreign exchange rate in Germany, Britain, Japan, and the United States over the period between January 1980 and December 2000. The data on industrial production and consumer price index are extracted from OECD publications. The short-term interest rate is one-month Euro rate. The exchange rates are expressed as the U.S. dollar price of the foreign currencies. The data on the Euro rates and the exchange rates are obtained from Datastream.

Using the maximum likelihood method, we estimate the seven-variable VAR (27) separately for three pairs of countries: U.S./Germany, U.S./U.K., and U.S./Japan. In each case, the variables included in $z_t$ are the growth rates of the U.S. and foreign industrial production ($y_t, y_t^*$), the U.S. and foreign rates of inflation ($\pi_t, \pi_t^*$), the U.S. and foreign one-month Euro rates ($i_t, i_t^*$), and the change in the exchange rate ($\Delta \ln S_t$).

3.1. The Estimated Monetary Policy Behavior

The identification restrictions imposed on matrix $C$ in section 2.3 imply that the U.S. and the foreign monetary authorities react contemporaneously to various economic shocks according to (abstracting from all lagged variables)

$$i_t = a_1 \varepsilon_{y,t} + a_2 \varepsilon_{\pi,t} + a_3 \varepsilon_{m,t} + a_4 \varepsilon_{s,t} (29)$$

$$i_t^* = a_1^* \varepsilon_{y,t}^* + a_2^* \varepsilon_{\pi,t}^* + a_3^* \varepsilon_{m,t}^* + a_4^* \varepsilon_{s,t} (30)$$

where $\varepsilon_{y,t}, \varepsilon_{\pi,t}$ and $\varepsilon_{m,t}$ are exogenous shocks to the U.S. output growth, inflation and monetary policy, respectively, while $\varepsilon_{y,t}^*, \varepsilon_{\pi,t}^*$ and $\varepsilon_{m,t}^*$ are exogenous shocks
TABLE 1. Estimates of the U.S. monetary policy reaction function

<table>
<thead>
<tr>
<th></th>
<th>U.S./Germany data</th>
<th>U.S./U.K. data</th>
<th>U.S./Japan data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.0138</td>
<td>0.0161</td>
<td>0.0158</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.0085</td>
<td>0.0088</td>
<td>0.0083</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.0484</td>
<td>-0.0250</td>
<td>0.0555</td>
</tr>
</tbody>
</table>

(0.0064) (0.0052) (0.0053) (0.0064) (0.0049) (0.0085) (0.0061) (0.0039) (0.0059)

Note: The reported figures are the estimates of the contemporaneous reactions of the U.S. monetary policy to various exogenous macroeconomic shocks. The figures in parentheses are the quasi-maximum likelihood standard errors. Abstracting from all lagged variables, the monetary policy reaction function is $i_t = a_1 \varepsilon y,t + a_2 \varepsilon \pi,t + a_3 \varepsilon m,t + a_4 \varepsilon s,t$, where $i_t$ is the short-term interest rate, $\varepsilon y,t$, $\varepsilon \pi,t$, $\varepsilon m,t$ and $\varepsilon s,t$ are exogenous shocks to the U.S. output growth, inflation, monetary policy, respectively. $\varepsilon s,t$ is an exogenous shock to the exchange rate.

to the corresponding foreign variables. Shock $\varepsilon s,t$ is an exogenous innovation to the exchange rate. The estimates of the contemporaneous policy reaction coefficients $a_i$ and $a^*_i$ ($i = 1, 2, 4$) are presented in Tables 1 and 2.

We can see from Table 1 that across all country pairs, the estimates of $a_1$ and $a_2$ are positive and highly significant, indicating that the Fed raises the short-term interest rate in response to positive innovations in output growth and inflation, which is consistent with the conventional view of the counter-cyclical monetary policy pursued by the Fed during that period. Also note that although the model is estimated for three different pairs of countries separately, the estimates are very close to each other.

Table 2 reports the corresponding policy reactions to the exogenous output and inflation shocks in the foreign countries. Although most coefficients are still positive, they are not significant, suggesting that the policy reactions in those countries are not as strong as in the United States.

It is interesting to note from Tables 1 and 2 that the estimate of $a_4$ and $a^*_4$ are highly significant in all cases, suggesting that there appear to be strong contemporaneous monetary policy responses in all countries to the exchange rate movements. This seems puzzling because the consensus is that the Fed does not react to movements in the exchange rate of the dollar, and such a recursive identification scheme has been widely used in the applications of VAR to study

TABLE 2. Estimates of foreign monetary policy reaction function

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>U.K.</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^*_1$</td>
<td>0.0011</td>
<td>0.0052</td>
<td>0.0008</td>
</tr>
<tr>
<td>$a^*_2$</td>
<td>0.0020</td>
<td>-0.001</td>
<td>-0.0006</td>
</tr>
<tr>
<td>$a^*_4$</td>
<td>-0.0218</td>
<td>-0.0415</td>
<td>0.0199</td>
</tr>
</tbody>
</table>

(0.0022) (0.0036) (0.0003) (0.00047) (0.0025) (0.0035) (0.0199)

Note: The reported figures are the estimates of the contemporaneous reactions of the foreign monetary policies to various exogenous macroeconomic shocks. The figures in parentheses are the quasi-maximum likelihood standard errors. Abstracting from all lagged variables, the monetary policy reaction function is $i^*_t = a^*_1 \varepsilon^*_y,t + a^*_2 \varepsilon^*_\pi,t + a^*_3 \varepsilon^*_m,t + a^*_4 \varepsilon^*_s,t$, where $i^*_t$ is the foreign short-term interest rate, $\varepsilon^*_y,t$, $\varepsilon^*_\pi,t$, $\varepsilon^*_m,t$ and $\varepsilon^*_s,t$ are exogenous shocks to the foreign output growth, inflation and monetary policy, respectively. $\varepsilon^*_s,t$ is an exogenous shock to the exchange rate.
the monetary policy effect on the exchange rate such as Eichenbaum and Evans (1995). One of the main differences between the present paper and the earlier VAR studies is that, instead of focusing on the United States alone, we included variables from both the United States and a foreign country in the VAR system. Therefore, the contemporaneous monetary policy responses to the exchange rate found in the current paper might simply reflect the correlation between the United States and the foreign short-term interest rates [see equations (29) and (30)].

3.2. The Exchange Rate Volatility

The Appendix shows that the exchange rate is determined by the following equation:

\[
\Delta \ln S_{t+1} = z_t' A_1 B_S A_2 z_t + (i_t - i_t^*) + (C_S z_t)' \varepsilon_{t+1},
\]

where \(B_S\) and \(C_S\) are, respectively, 4 \(\times\) 4 and 7 \(\times\) 7 matrices, and \(A_1\) and \(A_2\) are defined in Appendix. Under the symmetry assumption, the matrix \(C_S\) has a simple structure with only six unknown coefficients. This equation implies that the conditional variance of \(\Delta \ln S_{t+1}\) is given by \(z_t' C_S' C_S z_t\), where the estimates of the elements in \(C_S\) are reported in Table 3.

Consistent with many previous studies (e.g., Baillie and Bollerslev, 2000), we find that stochastic volatility is an important character of the foreign exchange rate movements, given the significant estimates of some of the elements in \(C_S\). In particular, we can see from Table 3 that, among all the estimates, \(C_{55}\) seems to be the statistically most significant and economically important element. Because \(C_{55}\) is the coefficient on \((i_t \varepsilon_{m,t+1} - i_t^* \varepsilon_{m,t+1})\) in the expression of \((C_S z_t)' \varepsilon_{t+1}\), this implies that the most important component of the conditional variance of the exchange rate movement, \(\Delta \log S_{t+1}\), is \(C_{55}^2 (i_t^2 \sigma_m^2 + i_t^* \sigma_{m * }^2)\), where \(\sigma_m^2\) and \(\sigma_{m * }^2\) are the variance of the exogenous interest rate shocks, \(\varepsilon_{m,t+1}\) and \(\varepsilon_{m,t+1}^*\), respectively. The higher the level of the interest rates or the larger the variance of the interest rate shocks, the more volatile is the exchange rate.

### Table 3. Estimates of the exchange rate volatility

<table>
<thead>
<tr>
<th></th>
<th>U.S./Germany Ex-rate</th>
<th>U.S./U.K. Ex-rate</th>
<th>U.S./Japan Ex-rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{11})</td>
<td>-0.0584 (0.0781)</td>
<td>-0.1354 (0.1648)</td>
<td>0.1121 (0.1163)</td>
</tr>
<tr>
<td>(C_{33})</td>
<td>0.5164 (0.4890)</td>
<td>-0.3148 (0.3706)</td>
<td>-0.9315 (0.5228)</td>
</tr>
<tr>
<td>(C_{55})</td>
<td>-5.8948 (0.3223)</td>
<td>-3.7672 (0.2900)</td>
<td>7.0690 (0.4663)</td>
</tr>
<tr>
<td>(C_{17})</td>
<td>0.0390 (0.0429)</td>
<td>-0.0539 (0.0628)</td>
<td>0.0191 (0.0542)</td>
</tr>
<tr>
<td>(C_{37})</td>
<td>0.0297 (0.0602)</td>
<td>-0.1472 (0.0717)</td>
<td>-0.0227 (0.0549)</td>
</tr>
<tr>
<td>(C_{57})</td>
<td>0.0076 (0.0689)</td>
<td>-0.0756 (0.0573)</td>
<td>-0.0540 (0.0551)</td>
</tr>
</tbody>
</table>

Note: The reported figures are the estimates of the elements in the matrix \(C_S\), whose definition can be found in the Appendix. The figures in parentheses are the robust standard errors. Under the symmetry assumptions, \(C_S\) has 6 unknown elements. \(C_{ij}\) represent the element in the \(i\)th row and \(j\)th column of the matrix. The exchange rate equation is given by \(\Delta \ln S_{t+1} = z_t' A_1 B_S A_2 z_t + (i_t - i_t^*) + (C_S z_t)' \varepsilon_{t+1}\). Hence, \(C_S\) determines the conditional variance of \(\Delta S_{t+1}\), which can be obtained as \(z_t' C_S C_S z_t\).
3.3. Time-Varying Currency Risk Premia

Equation (31) also reveals that the UIP deviation $\phi_t$ as defined in (13) is given by $z_t' A_1' B S A_2 z_t$. Therefore, following the discussion in Section 2.1, the currency risk premium $u_t$ defined in (15) can be expressed as:

$$u_t = z_t' A_1' B S A_2 z_t + \frac{1}{2} z_t' C_S C_S z_t. \quad (32)$$

As pointed out by Fama (1984), the negative slope coefficient from the linear regression of the change in the exchange rate on the interest rate differential implies that the risk premium must be negatively correlated with the exchange rate movement, and must be more volatile than the expected changes in the exchange rate, that is:

$$\text{corr}(\Delta \ln S_{t+1}, -u_t) < 0 \quad (33)$$

$$\text{std}(E_t \Delta \ln S_{t+1}) < \text{std}(u_t). \quad (34)$$

Table 4 summarizes the corresponding standard deviations and the correlation coefficients. We find that our parameterization of the market price of risk indeed produces foreign exchange risk premia with the requisite properties. The risk premia from investing in the German Mark, British Pound, and Japanese Yen are found to be negatively correlated with the subsequent change in the exchange rates and are more volatile than the expected changes in the exchange rates.

We also plot the estimated risk premium $u_t$ together with the interest rate differential $i_t - i^*_t$ and Jensen’s inequality term $v_t$ for the U.S./Germany, U.S./U.K., and U.S./Japan, respectively in Figures 1, 2, and 3. Consistent with previous results (e.g., Bekaert and Hodrick, 1992), the graphs confirm that, compared to the risk premia, Jensen’s inequality term is not an important factor affecting the exchange rate movements. Moreover, the graphs also clearly show that the risk premia are more volatile than the interest rate differential, implying that a large fraction of the exchange rate movements must be attributable to the changes in risk premia. Knowing how macroeconomic shocks affect the risk premia, therefore, may be critical for understanding the dynamics of exchange rate movements.

| TABLE 4. Currency risk premia and the exchange rate |
|-----------------------------------------|-----------------|-----------------|-----------------|
|                                         | U.S./Germany    | U.S./U.K.       | U.S./Japan      |
| std($u_t$)                            | 0.0112          | 0.0055          | 0.0107          |
| std($E_t \Delta \ln S_{t+1}$)         | 0.0106          | 0.0035          | 0.0102          |
| corr($\Delta \ln S_{t+1}, -u_t$)      | $-0.2245$       | $-0.1943$       | $-0.2236$       |

*Note:* This table summarizes the standard deviations of the estimated risk premium $u_t$ and the expected change in the exchange rate $E_t \Delta \ln S_{t+1}$, as well as the correlation coefficient between the risk premium and the subsequent exchange rate movement.
3.4. Variance Decomposition

One key aspect of the exchange rate movements that dynamic international asset pricing models fail to match is the volatility of the currency risk premia. Most reasonably parameterized models are found unable to produce the risk premia that are volatile enough to explain the UIP deviations when subject to usual macroeconomic shocks. Such failure could either reflect mis-specifications of
the models (such as investor’s preference) or some other exogenous shocks to the exchange rate not appropriately taken into account by economists. The VAR system discussed in the last section allows us to examine the sources of the volatility of the risk premia by imposing little restrictions on the structure of the economy.

Specifically, we calculate a variance decomposition for the foreign exchange risk premia analogous to those in linear VAR models based on Monte Carlo simulations. Random shocks ($\epsilon_{t+j}$, $j = 1, \ldots, 12$) to the VAR system (27) are drawn from a multivariate normal distribution and the 12-month forecasting errors for the foreign exchange risk premia $u_t$ are computed using (32). This process is repeated 500 times and the sample variances of the forecast errors at each time horizon ($j = 1, \ldots, 12$) are computed. In order to obtain a variance decomposition, each element of $\epsilon_{t+j}$ [see equation (21)] is drawn separately while holding all other elements fixed at zero during the simulations. The ratios of the variance due to each component of $\epsilon_{t+j}$ over the total variance then gives the variance decomposition. However, unlike linear VAR models, the sample variances from the Monte Carlo simulations are dependent on the past history $z_{t-1}$ due to the nonlinear restrictions imposed on the exchange rate movement. Therefore, we first calculate the variance decomposition conditional on each observation of $z_{t-1}$ in our sample over 1980 to 2000, and then take the average across $z_{t-1}$. The results are reported in Table 5.

In contrast to the results from the previous literature, we find that most of the volatilities of the currency risk premia can be accounted for by the identified standard macroeconomic shocks. In the case of the U.S./Germany exchange rate, output shocks and inflation shocks account for about 64% of the risk premium’s volatilities, and shocks to the monetary policies in the two countries account

TABLE 5. Variance decomposition for the currency risk premia

<table>
<thead>
<tr>
<th></th>
<th>Output shock</th>
<th>Inflation shock</th>
<th>Monetary policy shock</th>
<th>Exchange rate shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S./Germany exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td>0.2081</td>
<td>0.6215</td>
<td>0.1272</td>
<td>0.0431</td>
</tr>
<tr>
<td>6 month</td>
<td>0.2083</td>
<td>0.4555</td>
<td>0.1931</td>
<td>0.1432</td>
</tr>
<tr>
<td>12 month</td>
<td>0.2139</td>
<td>0.4333</td>
<td>0.1951</td>
<td>0.1577</td>
</tr>
<tr>
<td><strong>U.S./U.K. exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td>0.0502</td>
<td>0.0132</td>
<td>0.7491</td>
<td>0.1875</td>
</tr>
<tr>
<td>6 month</td>
<td>0.2333</td>
<td>0.2311</td>
<td>0.2932</td>
<td>0.2423</td>
</tr>
<tr>
<td>12 month</td>
<td>0.2379</td>
<td>0.2367</td>
<td>0.2805</td>
<td>0.2448</td>
</tr>
<tr>
<td><strong>U.S./Japan exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td>0.7674</td>
<td>0.1876</td>
<td>0.0210</td>
<td>0.0239</td>
</tr>
<tr>
<td>6 month</td>
<td>0.4913</td>
<td>0.2179</td>
<td>0.1443</td>
<td>0.1465</td>
</tr>
<tr>
<td>12 month</td>
<td>0.4488</td>
<td>0.2235</td>
<td>0.1632</td>
<td>0.1644</td>
</tr>
</tbody>
</table>

Note: This table reports the results of variance decomposition for the foreign exchange risk premia. The figures are the fractions of the total volatilities, up to 12 months ahead, of the risk premia accounted for by the exogenous shocks to output growth, inflation, monetary policy, and the exchange rate, respectively.

for another 20% of the volatilities. Similar results are found for the U.S./Japan exchange rate, in which the shocks to output, inflation, and monetary policies together account for nearly 85% of the risk premium’s volatilities, whereas the exogenous shocks to the exchange rate account for about 16% of the volatilities. In the case of the U.S./U.K. exchange rate, the exogenous exchange rate shocks account for a little larger fraction of the volatilities of the currency risk premium but still less than 25% of its total standard deviation. Moreover, among those standard macroeconomic shocks, the output and inflation shocks seem to be the most important ones, accounting for 64% (U.S./Germany), 46% (U.S./U.K.) and 67% (U.S./Japan) of the risk premium’s volatilities.

Moreover, our results suggest that the exogenous shocks to the monetary policies are an important force driving the foreign exchange risk premia, accounting for 16% (U.S./Japan) to 28% (U.S./U.K.) of the risk premium’s volatilities. To fully understand the dynamics of the exchange rate movement, therefore, it is crucial to explicitly model the policy behavior in the asset pricing models and investigate the mechanisms through which monetary policies affect the exchange rate.

We also compute the variance decompositions for output and inflation using the U.S./German, U.S./U.K., and U.S./Japan data, respectively. The results are reported in Tables 6 and 7. In all three cases, we find that almost all of the
TABLE 6. Variance decomposition for the U.S. output growth

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output shock</td>
<td>Inflation shock</td>
<td>Monetary policy shock</td>
</tr>
<tr>
<td>1 month</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>6 month</td>
<td>0.9843</td>
<td>0.0016</td>
<td>0.0115</td>
</tr>
<tr>
<td>12 month</td>
<td>0.9680</td>
<td>0.0040</td>
<td>0.0227</td>
</tr>
</tbody>
</table>

Note: This table reports the results of variance decomposition for the industrial output growth rate in the United States. The figures are the fractions of the total volatilities, up to 12 months ahead, of the output growth rate accounted for by the exogenous shocks to output growth, inflation, monetary policy, and the exchange rate, respectively.

TABLE 7. Variance decomposition for the U.S. inflation rate

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output shock</td>
<td>Inflation shock</td>
<td>Monetary policy shock</td>
</tr>
<tr>
<td>1 month</td>
<td>0.0246</td>
<td>0.9754</td>
<td>0.0000</td>
</tr>
<tr>
<td>6 month</td>
<td>0.0933</td>
<td>0.8646</td>
<td>0.0275</td>
</tr>
<tr>
<td>12 month</td>
<td>0.1045</td>
<td>0.8098</td>
<td>0.0555</td>
</tr>
</tbody>
</table>

Note: This table reports the results of variance decomposition for the CPI inflation rate in the United States. The figures are the fractions of the total volatilities, up to 12 months ahead, of the inflation rate accounted for by the exogenous shocks to output growth, inflation, monetary policy, and the exchange rate, respectively.
volatilities in output are accounted for by the output shocks (more than 95% on
average). The inflation shocks and the monetary policy shocks account for another
2 to 3% of its volatilities. The shocks to the exchange rate only account for less than
2% of the volatilities of output. Similar results are found for inflation as well. All
standard macroeconomic shocks together account for nearly 95% of its volatilities
with the inflation shocks being the most important ones. The exchange rate shocks
only account for about 5% of the volatilities of inflation. These results confirm that
the foreign exchange risk premia are driven mostly by the same macroeconomic
shocks that affect output and inflation.14

The standard representative agent models fail to account for the forward pre-
mium anomaly in currency market mainly because they impose a very close link
between the real exchange rate and aggregate consumption. Introducing frictions
in the goods market such as sticky prices (e.g., Chari, Kehoe, and McGrattan,
2003) or in the asset market such as transaction costs (e.g. Alvarez, Atkeson, and
Kehoe, 2002), international business cycle models can break such a tight link.
When subject to the standard macroeconomic shocks, they are able to generate
volatile and persistent real exchange rates, eliminating some discrepancies between
the theoretical models and the data. The empirical results obtained in our paper
complement these theoretical studies, and suggest that a promising approach to
understand the anomaly in currency markets is to extend the standard international
asset pricing models to include various imperfections in the goods and/or asset
markets.

3.5. Exchange Rate Overshooting

Economists have long recognized the importance of monetary policy shocks for
the movement of exchange rates. The well-known Dornbush (1976) overshooting
model predicts that the exchange rate will initially overshoot its long-run level
in response to an exogenous monetary shock that alters the domestic and foreign
interest rate differential. However, as seen in Figures 1–3, the difference between
the volatilities of risk premia and interest rate differentials is wide enough to
suggest that a large fraction of exchange rate movements must be attributable
to the changes in risk premia. Knowing how monetary policy shocks affect risk
premia, therefore, may be critical for understanding the dynamics of exchange
rate movements under such shocks.

In this section, we revisit the issue of exchange rate overshooting. In particular,
we examine the following two questions: (i) how does an exogenous monetary
policy shock affect the currency risk premium? and (ii) How does the response of
the risk premium to the policy shock affect exchange rate movements?

In particular, we follow the literature on nonlinear impulse response functions
(IRF) and calculate the dynamic responses of $z_t$ under the macroeconomic shock
$\varepsilon_t$ as the difference between a pair of conditional expectations:15

$$E(z_{t+h} \mid \Omega_{t-1}, \varepsilon_t) - E(z_{t+h} \mid \Omega_{t-1}).$$
FIGURE 4. IRFs of macroeconomic variables. (This figure plots the impulse-response functions of macroeconomic variables under an exogenous monetary policy shock of the size of 1 standard deviation using U.S./German data.)

where $\Omega_{t-1}$ stands for the information set (or the state) at $t - 1$ (including $z_{t-1}$), and $h = 0, 1, 2, \ldots$ is time horizon.

In this paper, we consider an exogenous shock to the U.S. monetary policy that pushes down the U.S. short-term interest rate relative to the foreign rate. We compute the IRF conditional on each observation of $\Omega_{t-1}$, denoted by $\omega_{t-1}$, between January 1980 and December 2000. Assuming stationarity, these IRFs conditional on $\omega_{t-1}$ are realizations of the random variables defined by these conditional expectations. To calculate the expectations conditional on $\omega_{t-1}$, we simulate the model in the following manner. First, we fix $\omega_{t-1}$ and randomly draw $\varepsilon_{t+j}$ from $\mathcal{N}(0, I)$ for $j = 1, 2, \ldots, h$ and then simulate the model conditional on $\omega_{t-1}$ and monetary policy shock $\varepsilon_t$. This process is repeated 500 times and the estimated conditional expectation is obtained as the average of the outcomes.

Figures 4–6 display the estimated IRFs of output growth, inflation, and the short-term interest rate in the home and foreign country using the U.S./Germany,
U.S./U.K., and U.S./Japan data, respectively. The IRFs are computed conditional on each realization of $\Omega_{t-1}$ between January 1980 and December 2000 under an exogenous expansionary shock to U.S. monetary policy. Each line in the graphs corresponds to a particular realization of the random impulse response function defined earlier. We can see that although there are some variations in the effects of the policy shock on these variables due to nonlinearity in the VAR model, the IRFs are very similar to those frequently reported in the standard monetary VAR literature. In particular, under the expansionary monetary policy shock, the U.S. short-term interest rate falls, which in turn leads to a mild decline in the foreign interest rate as the foreign monetary authority reacts to the U.S. monetary actions. These declines in interest rates eventually lead to increases in output in both the United States and the foreign country through the usual monetary transmission mechanisms. By contrast, the inflation rates in the United States and the foreign country appear to fall in response to the expansionary monetary shock, similar to
the widely observed phenomenon dubbed as the “price puzzle” in the standard monetary VAR literature.

Figure 7 reports the responses of risk premia from investing in three currencies to the U.S. monetary policy shock. The left side panels display the IRFs conditional on different $z_{t-1}$ all together, whereas the right side panels show their averages. It is easy to see that the shocks to U.S. monetary policy have a significant impact on risk premia. On average, an expansionary shock to U.S. monetary policy generates a large increase in the currency risk premium. In the case of the German Mark, the risk premium increases by 20 basis points on average, whereas the risk premia on the British Pound and Japanese Yen increase on average by 11 and 5 basis points, respectively, in response to a monetary policy shock of one standard deviation.

Such a positive response of the foreign exchange risk premium to an expansionary monetary shock is consistent with the standard economic models. The foreign exchange risk premium is, by definition, the expected excess return from investing
in a foreign currency under risk aversion. An expansionary monetary shock not only lowers the U.S. interest rate relative to the foreign rate but also leads to an expected depreciation of the U.S. dollar or an expected appreciation of the foreign currency as most economic models predict.

More important, the responses of the currency risk premium to the monetary policy shock vary substantially across different states of the economy (or different initial values of $z_{t-1}$). The responses range from a slightly negative number to positive 1.6% in the case of the German Mark, and from less than 5 to nearly 35 basis points in the case of the British Pound. These large variations have important implications for exchange rate movements in response to the monetary shocks.

According to Dornbush’s (1976) overshooting mechanism, the exchange rate initially overshoots its long-run level in response to an expansionary monetary shock. In this mechanism, interest rates play a central role in affecting the dynamics...
of exchange rate movements. However, in the presence of time-varying risk premia, the exchange rate movement is determined by the risk premia \( u_t \) as much as by the interest rate differential \( i_t - i^*_t \), as is observed in (15). Indeed, many empirical studies find that, instead of immediate overshooting, we often observe persistent dollar depreciations before it starts to move to its long run level in response to the U.S. expansionary monetary policy shocks (e.g., Eichenbaum and Evans, 1995). Although many economists try to rationalize such delayed overshooting based on the dynamics of interest rate movements, this phenomenon is completely consistent with the existence of volatile currency risk premia and their responses to the monetary shocks, as shown in Figure 7.

More specifically, although an expansionary shock to U.S. monetary policy lowers the U.S. interest rate relative to the foreign rate (see Figures 4–6), it also raises the risk premium. The lower U.S. interest rate makes the foreign currency more attractive and leads to a depreciation of the U.S. dollar. If there were no risk premia or the risk premia were constant, equation (13) (ignoring Jensen’s
inequality term) would imply that there should be a subsequent appreciation of the U.S. dollar relative to the foreign currency following the initial reaction of the exchange rate due to international arbitrage. Hence, in such a case, the exchange rate must initially overshoot its long run level. However, in the presence of time-varying risk premium, the movement in the exchange rate depends on both the risk premium and the interest rates, and, in particular, on the magnitude of the response of the risk premium. If $u_t$ increases more than a decline in $(i_t - i_t^*)$ in response to the monetary shock, the dollar would continue to depreciate as dictated by the risk-premium-adjusted UIP given in (13) and, therefore, exhibits the “delayed overshooting.” By contrast, if $u_t$ increases less than a decline in $(i_t - i_t^*)$ in response to the monetary shock, then the exchange rate will behave according to the standard overshooting mechanism.

In Figure 8, we plot two typical cases of the responses of the exchange rate following an expansionary shock to the U.S. monetary policy. The upper-left panel of the figure presents the IRF of the exchange rate when the response of the risk premium is larger than that of the interest rate differential (shown in the lower-left panel) under the monetary shock. The upper-right panel of the figure displays the IRF of the exchange rate when the response of the risk premium is smaller than that of the interest rate differential (shown in the lower-right panel). Both IRFs are drawn conditional on particular actual historical dates. We can clearly see how the dynamics of exchange rate movements depend on the size of the response of risk premia to the monetary shock.

4. CONCLUDING REMARKS

In this paper we empirically examine the sources of volatilities of the foreign exchange risk premia. The study is motivated by the observation that reasonably parameterized international asset pricing models fail to generate the currency risk premia volatile enough to match the empirical evidence. Using a nonlinear structural VAR based on the no-arbitrage condition to identify various fundamental shocks and the foreign exchange risk premia, we find that most of the volatilities of the foreign exchange risk premia can be accounted for by the standard macroeconomic shocks that drive output and inflation. Exogenous shocks to the foreign exchange markets have a small, although not negligible, impact on the foreign exchange risk premia.

If the foreign exchange risk premia are time-varying and volatile, then a large fraction of the movement in the exchange rate must be attributed to the fluctuations in the risk premium. Therefore, knowing the behavior of the risk premia may be crucial to understand the dynamics of the exchange rate movements in response to exogenous macroeconomic shocks. Our findings help reconcile the seemingly contradicting observations from previous VAR analysis of the exchange rate movement under exogenous monetary innovations.

One important question that is left unanswered in this work is that why the currency risk premium or the exchange rate is so volatile. Many studies have
proposed various explanations based on structural models. The empirical results of the current paper provide some insight into the dynamic effects of different macroeconomic shocks as an intermediate step toward bridging the gap between economic theories and the empirical evidence.

NOTES

1. Other explanations of the failure of UIP include irrational expectations, “peso problems,” learning, statistical artifact, monetary policy impact, etc. See papers discussed in Lewis (1995). In this work, we take the perspective that the deviations from UIP reflect the presence of rational, time-varying risk premia.

2. This is similar to the failure of consumption-based asset pricing models in the equity premium puzzle literature.

3. The critical component of the exchange rate overshooting mechanism is uncovered interest rate parity. If a time-varying risk premium is also present, we then have $E_t \Delta \ln S_{t+1} = (i_t - i_t^*) + \phi_t$, where $\phi_t$ is the risk premium. Fama (1984) argues that $\phi_t$ is extremely volatile. Therefore, the movement in the exchange rate $\Delta \ln S_{t+1}$ is affected as much by $\phi_t$ as by the short-term interest rates $i_t$ and $i_t^*$.

4. Note that if markets are complete, there will be unique $M_{t+1}$ and $M_{t+1}^*$. Otherwise, we can interpret $M_{t+1}^*$ and $M_{t+1}$ as the minimum variance discount factors and hence are unique (Cochrane 2001). In either case, we can define $M_{t+1}^*$ to be $M_{t+1} \frac{S_{t+1}}{S_t}$.

5. Consider a one-period risk-free bond, (1) and (3) imply, respectively, that $e^{-i_t} = E_t (M_{t+1}^*)$ and $e^{-i_t^*} = E_t (M_{t+1})$.

6. Note that similar results hold for the foreign country as well. The currency risk premium for foreign investors can be expressed as $\mu_t^* = \sum_{i=1}^{N+1} \lambda_{i,t}^* \cdot \text{Cov}_t [-\Delta \ln S_{t+1} - (i_t^* - i_t), \varepsilon_{i,t+1}]$, and the similar interpretation applies to $\lambda_{i,t}^*$.

7. We can easily generalize the model to include more economic shocks.

8. Similar parameterizations of the market price of risk have been widely used in the literature where $z_t$ is treated as a set of latent state variables, including Constantinides (1992), Ahn et al. (2002), and Dai and Singleton (2002), among many others.

9. Our empirical exercises that follow are based on the point estimates of the parameters in the model. An important extension might be to take into account parameter uncertainties, including the problem of choosing the lag length.

10. We identify the policy response to the exchange rate shock by assuming that the U.S. (foreign) monetary authority only responds to the U.S. (foreign) output and inflation shocks. See the zero restrictions imposed on the last two rows of matrix $C$ in Section 2.3.

11. Interpretation of the sign of the estimates of $\alpha_k$ and $\alpha_k^*$ needs some caution. A negative estimate of the U.S. monetary policy response to an exogenous exchange rate shock does not necessary mean that the Fed seeks to cut the short term interest rate when the U.S. dollar is depreciating, because the direction of movement of the exchange rate $\Delta \ln S_t$ depends on $C_t z_{t-1}$ [see equation (28)].

12. The unconditional correlation coefficient between the one-month Euro rates, $i_t$ and $i_t^*$, ranges from 0.52 for U.S./Germany to 0.70 for U.S./U.K.

13. Note that the foreign exchange risk premium defined in this paper has the opposite sign as that in Fama (1984), which defines the risk premium as $(i_t - i_t^*) - E_t (\Delta \ln S_{t+1})$.

14. As a simple measure for the amount of nonlinearity, we estimated a standard linear VAR by ignoring the restrictions in the last equation of the system, and compare the resulting variance decompositions to the figures reported in Tables 5–7. The sharp contrasts we observed in our nonlinear VAR models tend to lessen when the linear VAR models are used. For example, the output and monetary policy shocks account for 93% and 1.4%, respectively, of the volatility of the U.S. output based on U.S./Japan data in our nonlinear model. The ratios become 86% and 7.7%, respectively, in the linear model. Even bigger differences are found for the foreign exchange risk premia. The exogenous
exchange rate shock accounts for less than 24.5% of the volatility of the U.S./U.K. currency risk premia in our nonlinear model, whereas the ratio becomes to 38% in the linear model.

15. See Koop et al. (1996), Gallant et al. (1993), and Potter (2000), among others.

16. Only an expansionary monetary policy shock is considered here.

17. For example, Gourichas and Tornell (1996) argue that as the market cannot distinguish between the persistent component and the transitory component of interest rate shocks, the delayed overshooting results from the interaction of learning by the market and the dynamic response of interest rates to monetary shocks.

18. We have ignored the term \( (i_{t-1} - i^*_t) \) here, as it does not affect the conditional covariance.

REFERENCES


APPENDIX

The first identifying restriction is based on the assumption that home and foreign investors price the currency risk in a symmetrical fashion in the sense described as follows. For example, let us consider the first two elements of $\varepsilon_t$: the shocks to the home and foreign country’s output ($\varepsilon_{y,t}$ and $\varepsilon^*_{y,t}$). To investors in the home country, the currency risk associated with the shock to home output is $\text{Cov}_{t-1}\left[\Delta \ln S_y, \varepsilon_{y,t}\right]$, whereas to investors in the foreign country the currency risk associated with foreign output is $\text{Cov}_{t-1}\left[-\Delta \ln S_y, \varepsilon^*_{y,t}\right]$. We assume that if the market price for the risk (or the expected excess rate of return per unit of the covariance) in the home country is given by:

$$
\lambda_{1,t} = \Gamma_{11}y_t + \Gamma_{12}y^*_t + \Gamma_{13}\pi_t + \Gamma_{14}\pi^*_t + \Gamma_{15}i_t + \Gamma_{16}i^*_t + \Gamma_{17}\Delta \ln S_y,
$$

then the foreign counterpart is given by:

$$
\lambda^*_2,t = \Gamma_{12}y_t + \Gamma_{11}y^*_t + \Gamma_{14}\pi_t + \Gamma_{15}\pi^*_t + \Gamma_{16}i_t + \Gamma_{15}i^*_t - \Gamma_{17}\Delta \ln S_y,
$$

where $\Gamma_{ij}$ refers to the element on the $i$th row and $j$th column of matrix $\Gamma$. And similar parameterizations apply to $\lambda_{2,t}$ and $\lambda^*_{2,t}$, for $i = 2, \ldots, 6$. For $\lambda_{7,t}$ and $\lambda^*_{7,t}$, notice that if the currency risk due to the exogenous exchange rate shock $\varepsilon_{S,t}$ is $\text{Cov}_{t-1}\left[\Delta \ln S_y, \varepsilon_{S,t}\right]$ for home investors, its foreign analogy is $\text{Cov}_{t-1}\left[-\Delta \ln S_y, -\varepsilon_{S,t}\right]$. Hence, the symmetric assumption implies that if $\varepsilon_{S,t}$ has a market price of risk in the home country given by:

$$
\lambda_{7,t} = \Gamma_{71}y_t + \Gamma_{72}y^*_t + \Gamma_{73}\pi_t + \Gamma_{74}\pi^*_t + \Gamma_{75}i_t + \Gamma_{76}i^*_t + \Gamma_{77}\Delta \ln S_y,
$$
then in the foreign country its market price of risk will be:

\[ \lambda^*_t = -\Gamma^*_t y_t - \Gamma^{**t}_t y^*_t - \Gamma^{**t}_{t1} \pi_t - \Gamma^{**t}_{t2} \pi^*_t - \Gamma^{**t}_{t3} \pi_{t1} - \Gamma^{**t}_{t4} \pi_{t2} - \Gamma^{**t}_{t5} \pi_{t3} - \Gamma^{**t}_{t6} \pi_{t4} - \Gamma^{**t}_{t7} i_t + \Gamma^{**t}_{t7} \Delta \ln S_t. \]

In summary, the symmetric treatment of the market price of risk across countries implies that \( \Gamma^* = A \Gamma A \)

where:

\[
A = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}.
\]

Pre- and postmultiplication of matrix \( A \) has an effect on matrix \( \Gamma \) in the following manner: First, it changes the position of the first and second rows, the third and fourth rows, and the fifth and sixth rows of matrix \( \Gamma \) and then changes the sign of the last row. Second, it changes the position of the first and second columns, the third and fourth columns, and the fifth and sixth columns of matrix \( \Gamma \) and then changes the sign of the last column.

With this restriction, the last equation in (27) can be expressed as:

\[ \Delta \ln S_t = z'_{t-1} A_1 B_3 A_2 z_{t-1} + (i_t - i^*_t) + (C_S z_{t-1})' \epsilon_t, \]

or:

\[ \Delta \ln S_t = z'_{t-1} A_1' B_3 A_2' z_{t-1} + b' z_{t-1} + (C_S z_{t-1})' \epsilon_t, \]

where \( B_3 \) and \( C_S \) are, respectively, \( 4 \times 3 \) and \( 7 \times 7 \) matrices whose elements are to be estimated, \( b = (0, 0, 0, 1, -1, 0)' \) as defined in (27), and the matrices \( A_1 \) and \( A_2 \) are given by:

\[
A_1 = \begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix} \quad \text{and} \quad A_2 = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0
\end{pmatrix}.
\]

To see this, note that \( z'(\Gamma' \Gamma - \Gamma'^* \Gamma^*)z = z'(\Gamma' \Gamma - A \Gamma A)z = z' \Gamma' \Gamma z - z' \Gamma \tilde{z} - \tilde{z}' \Gamma' \Gamma \tilde{z} = (z - \tilde{z})' (\Gamma' \Gamma)(z + \tilde{z}), \) where \( \tilde{z} = Az. \) Now:

\[
z - \tilde{z} = \begin{pmatrix}
z_1 - z_2 \\
z_2 - z_1 \\
z_3 - z_4 \\
z_4 - z_3 \\
z_5 - z_6 \\
z_6 - z_5 \\
2z_7
\end{pmatrix} \quad \text{and} \quad z + \tilde{z} = \begin{pmatrix}
z_1 + z_2 \\
z_2 + z_1 \\
z_3 + z_4 \\
z_4 + z_3 \\
z_5 + z_6 \\
z_6 + z_5 \\
0
\end{pmatrix}.
\]
Note also that:
\[
\begin{bmatrix}
  z_1 - z_2 \\
  z_3 - z_4 \\
  z_5 - z_6 \\
  z_7
\end{bmatrix}
= A_1 z \quad \text{and} \quad
\begin{bmatrix}
  z_1 + z_2 \\
  z_3 + z_4 \\
  z_5 + z_6
\end{bmatrix}
= A_2 z.
\]

Therefore, if there is no restriction on $\Gamma_1$, we can express the original quadratic form as
\[
z' (\Gamma' \Gamma - \Gamma'^* \Gamma^*) z = z' A_1' B_5 A_2 z
\] as claimed.

The second set of restrictions is based on another type of symmetric assumption to simplify the expression of matrix $C_S$. We assume, for example, the contribution of $y^*$ to the market price of home output risk is assumed to be equal in size to the contribution of $y$ to the market price of foreign output risk. This type of symmetric assumption implies restrictions on matrix $\Gamma$ in the form of $\Gamma_{1+2i,1+2j} = \Gamma_{2+2i,2+2j}$ and $\Gamma_{1+2k,2+2l} = \Gamma_{2+2k,1+2l}$ for $i, j, k, l = 0, 1, 2$ and $i \neq j$. It makes all off-diagonal elements of $C_S$ except the last row and the last column equal to zero. The resulting matrix $C_S$ becomes:
\[
C_S = \begin{pmatrix}
  C_{11} & 0 & 0 & 0 & 0 & 0 & C_{17} \\
  0 & -C_{11} & 0 & 0 & 0 & 0 & C_{17} \\
  0 & 0 & C_{33} & 0 & 0 & 0 & C_{37} \\
  0 & 0 & 0 & -C_{33} & 0 & 0 & C_{37} \\
  0 & 0 & 0 & 0 & C_{55} & 0 & C_{57} \\
  0 & 0 & 0 & 0 & 0 & -C_{55} & C_{57} \\
  C_{17} & C_{17} & C_{37} & C_{37} & C_{57} & C_{57} & 0
\end{pmatrix}.
\]